Introduction to Fractions

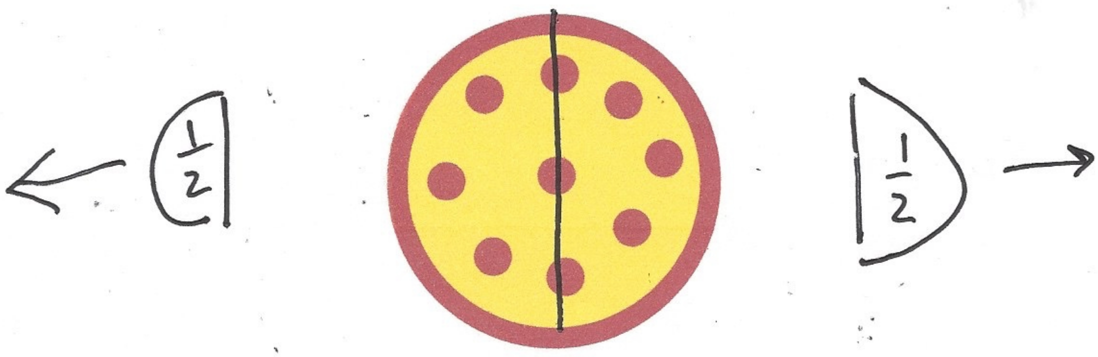
What is a fraction?

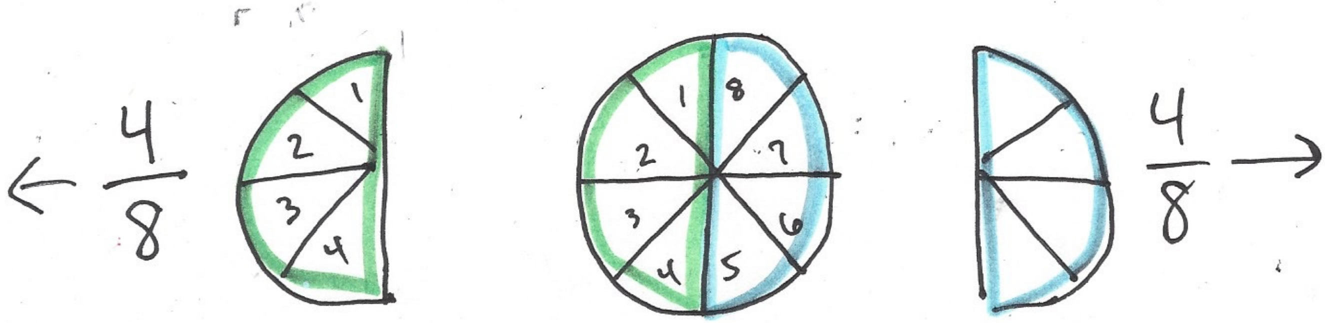
There are a lot of different ways to look at fractions, but we will primarily be looking at them as a way to describe something (a pizza, a number, something else) being divided up into equal parts.

For example, the fraction one half:

$$\frac{1}{2}$$

represents what happens when we take 1 pizza (the number on top) and divide it into 2 pieces (the number on the bottom):





The Parts of a Fraction

In this fraction, the top number is called the **numerator**, and the bottom number is called the **denominator**:

$$\frac{1}{2} = \frac{numerator}{denominator}$$

The line in the middle is known as the **fraction bar** or the **vinculum**, if you're being fancy.

How to Read a Fraction

The number ½ is read as "one half," but how do you pronounce the other fractions?

Generally, read the top number as the plain number, but then read the bottom number as an **ordinal number** (third, fourth, fifth, sixth, etc.)

Ex:

$\frac{1}{3}$ One third $\quad$ $\frac{4}{5}$ Four fifths

$\frac{1}{4}$ One fourth $\quad$ $\frac{17}{6}$ Seventeen sixths

The exception to this rule is when the denominator is 1 or 2:

$\frac{1}{2}$ One half $\quad$ $\frac{9}{2}$ Nine halves

$\frac{3}{1}$ Three over one $\quad$ $\frac{10}{1}$ Ten over one

Raising and Reducing Fractions

One useful fact about fractions is that you can multiply the numerator and denominator by the same value (other than 0) and the new fraction will be equal to the original fraction:

$\frac{1}{2} = \frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6}$ This process is called **raising terms**

Ex:

$$\frac{1}{2} = \frac{1 \cdot 5}{2 \cdot 5} = \frac{5}{10}$$

$$\frac{1}{2} = \frac{1 \cdot (-1)}{2 \cdot (-1)} = \frac{-1}{-2}$$

In the same way, you can also divide the numerator and denominator by the same value (other than 0.) This is called **reducing or cancelling**

$$\frac{5}{10} = \frac{\overset{1}{\cancel{5}} \cdot 1}{\underset{1}{\cancel{5}} \cdot 2} = \frac{1}{2}$$

$$\frac{-1}{-2} = \frac{\overset{1}{\cancel{(-1)}} \cdot 1}{\underset{1}{\cancel{(-1)}} \cdot 2} = \frac{1}{2}$$

Ex: Reduce these fractions:

$$\frac{15}{25} = \frac{3\cdot\cancel{5}^{1}}{5\cdot\cancel{5}_{1}} = \boxed{\frac{3}{5}}$$

$$\frac{15}{27} = \frac{\cancel{3}^{1}\cdot 5}{{}_{1}\cancel{3}\cdot 9} = \boxed{\frac{5}{9}} = \frac{5}{\cancel{3}\cdot 3}$$

$$\frac{100}{48} = \frac{\boxed{10}\cdot\boxed{10}}{\boxed{6}\cdot\boxed{8}} = \frac{\boxed{\cancel{2}^{1}\cdot 5}\cdot\boxed{\cancel{2}^{1}\cdot 5}}{\boxed{\cancel{2}_{1}\cdot 3}\,\boxed{\cancel{2}_{1}\cdot 4}} = \frac{5\cdot 5}{3\cdot 4} = \boxed{\frac{25}{12}}$$

Often, we use a shortcut to write out the reducing:

$$\frac{36}{81} = \overset{\text{Cancel 9}}{\frac{\cancel{36}^{4}}{\cancel{81}_{9}}} = \boxed{\frac{4}{9}} \quad \text{or} \quad \overset{\text{Cancel 3}}{\frac{\cancel{36}^{12}}{\cancel{81}_{27}}} = \overset{\text{Cancel 3}}{\frac{\cancel{12}^{4}}{\cancel{27}_{9}}} = \boxed{\frac{4}{9}}$$

$$\frac{36}{6} = \frac{\cancel{36}^{6}}{\cancel{6}_{1}} = \frac{6}{1} = ? = \boxed{6}$$

Another useful fact about fractions is that when the denominator is 1, the fraction is equal to an integer:

$$\frac{5}{1} = 5 \qquad \frac{1}{1} = 1$$

$$\frac{0}{1} = 0 \qquad \frac{-17}{1} = -17$$

You can also write integers in **fractional form** using this fact:

$$4 = \frac{4}{1} \overset{\cdot 2}{\underset{\cdot 2}{=}} \frac{8}{2} \overset{\cdot 3}{\underset{\cdot 3}{=}} \frac{24}{6} \overset{-1}{\underset{-1}{=}} \frac{-24}{-6}$$

$$-28 = \frac{-28}{1} \overset{\cdot 5}{\underset{\cdot 5}{=}} \frac{-140}{5} \overset{\cdot -1}{\underset{\cdot -1}{=}} \frac{140}{-5}$$